EXTENSION LECTURE

BY

Mr. N.S.V. KIRAN KUMAR
Lecturer in Mathematics
Department of Mathematics
GOVERNMENT DEGREE COLLEGE
MANDAPETA

TOPIC	**:**	**GROUPS**
VENUE	**:**	**GOVERNMENT DEGREE COLLEGE, RAVULAPALEM**
		Dr. B.R. AMBEDKAR KONASEEMA DISTRICT
DATE	**:**	**September 08, 2023**
TIME	**:**	**10 AM to 12 Noon.**

TOPIC SYNOPSIS

Binary operation: An operation 'o' is said to be binary on a non-empty set G if for all a, b ∊ Gthen a o b ∊ G.

Example:

1. Addition (+) is a binary operation on set of Natural Numbers 'N'.
2. Subtraction (-) is not a binary operation on N. Since for a = 5, b = 9 ∊ N then a + b = 5 + 9 = 14 ∊ N but a - b = 5 – 9 = – 4 ∉ N

Algebraic Structure: A non-empty set together with one or more than one binary operation is called analgebraic structure.

Examples:

1. (R, +), (R, ·) . is an Algebraic Structure where R is set of Real Numbers.
2. (N, +) , (Z, +), (Q, +) are algebraic structures but (N, –) (Z, ÷) are not an algebraicstructures.
3. Division (÷) is not a binary operations on Z . Since for a = 2, b = 3 ∈ Z but 2 ÷ 3 = $\frac{2}{3}$ ∉ Z. Therefore (Z, ÷) is not an Algebraic structure.
4. Multiplication is a Binary operation on the set or Rational numbers Q. For $a = \frac{2}{3}$ and $b = \frac{4}{7} \in Q$ then a · b = $\frac{2}{3} \cdot \frac{4}{7} = \frac{6}{21}$ ∈ Q. Therefore (Q, ·) is an Algebraic Structure.

Closure Property: $\forall\ a, b \in G \Rightarrow\ aob \in G$.

Example:

1. (N, +), (Z, +) are satisfies Closure Property.
2. (N, –) does not satisfies Closure Property. Since for 2 ∈ N and 3 ∈ N but 2 – 3 = – 1 ∉ N

Associative Property: $a\ o\ (\ \ c\ ) = (a\ o\ b)\ o\ c\ \ \forall\ a, b, c \in G$

Example: (N, +), (Z, +), (R, +), (Q, +) are satisfies Associative Property.

Identity Properties: Let S be a non-empty set and 'o' be a binary operation on S.

(i) If there exists and element $e_1 \in S$ such that e_1 o a = a for a ∈ S then 'e_1' is called a left identity of S w.r.t. the operation 'o'.

(ii) If there exists and element $e_2 \in S$ such that a o e_2 = a for a ∈ S then 'e_2' is called a right identity of S w.r.t. the operation 'o'.

(iii) If there exists and element e ∈ S such that e is both a left and right identity of S w.r.t. 'o' i.e. $a\ o\ e = e\ o\ a = a$, then 'e' is called an identity of S.

Example:

1. (Z, +) is satisfies Identity Property. Since ∀ a ∈ Z ∃ an element 0 ∈ Z such that a + 0 = 0 + a = a.
2. (N, +) is not satisfies identity Property. Since ∀ a ∈ N ∃ an element 0 ∉ N such that a + 0 = 0 + a = a.
3. But (N, ·) satisfies Inverse Property. Since ∀ a ∈ N ∃ an element 1 ∈ N such that a · 1 = 1 · a = a.

Inverse Property: Let S be a non-empty set and 'o' be a binary operation on S with identity element 'e'

(i) An element a ∈ S is said to be left invertible or left regular if there exists an element b ∈ S such that b o a = e. b is called a left inverse of 'a' w.r.t. 'o'.

(ii) An element a ∈ S is said to be right invertible or right regular if there exists an element b ∈ S such that a o b = e. b is called a right inverse of 'a' w.r.t. 'o'.

(iii) An element b ∈ S which is both a left inverse and right inverse of 'a' i.e. a o b = b o a = e is called an inverse of 'a' and 'a' is said to be invertible or regular.

Example:

1. (Z, +) is satisfies Inverse Property. Since ∀ a ∈ Z ∃ an element –a ∈ Z such that a + (–a) = (–a) + a = 0.
2. (N, +) is not satisfies Inverse Property. Since ∀ a ∈ N ∃ an element –a ∉ N such that a + (–a) = (–a) + a = 0.
3. (Z, ·), (N, ·) are not satisfies inverse property. Since 2 ∈ N and Z ∃ an element $\frac{1}{2} \notin$ N and ∉ Z such that $2 \cdot \frac{1}{2} = \frac{1}{2} \cdot 2 = 1$.
4. (R, ·) is not satisfies inverse property. Since 0 ∈ R ∃ an element $\frac{1}{0}$ is undefined. ∴ '0' has no multiplicative inverse. Similarly (Q, ·) is not satisfies inverse property.
5. R* is the set of non-zero real numbers i.e. R* = R – {0}. ∴ (R*, ·) is satisfies Inverse Property.

Abelian Property or Commutative Property: ∀ a, b ∈G ⇒ a o b = b o a

Example: (N, +), (Z, +), (Q, ·), (R, ·) are satisfies Abelian and Commutative Property.

Groupoid: A non – empty set G is said to be Groupoid w.r.t to the operation 'o' if it satisfies Closure Property.

Example:

1. (N, +), (Z, +) are satisfies Closure Property. ∴ (N, +), (Z, +) are Groupoid w.r.t. '+'
2. (N, –) does not satisfies Closure Property. Since for $2 \in N$ and $3 \in N$ but $2 - 3 = -1 \notin N$. ∴ (N, –) is not a Groupoid w.r.t. '–'

Semi-Group: A non – empty set G is said to be a semi-group w.r.t. operation 'o' if itsatisfies Closure Property and Associative Property.

Example:

1. (N, +), (Z, +), (R, ·), (Q, ·) are satisfies Closure and Associative Property.
   ∴ (N, +), (Z, +), (R, ·), (Q, ·) are Semi-group.

Monoid: A non – empty set G is said to be a Monoid w.r.t to the operation 'o' if it satisfies Closure Property, Associative Property and Identity Property.

Example:

1. (N, +) is not a Monoid. Since N satisfies Closure Property and Associative Property but not Identity Property i.e. Identity element w.r.t '+' is $0 \notin N$.
2. (N, ·) is Monoid.
3. (Z, ·), (Z, +), (R, ·), (R, +), (Q, ·), (Q, +) are Monoid

Group : A non-empty set G is said to be a Group w r t a Binary operation 'o' if itsatisfies the Closure Property, Associative Property, Identity Property and Inverse Property.

Example:

1. (Z, +), (R, +), (Q, +) are Group. But (Z, ·), (R, ·), (Q, ·) are not Group
2. (R*, ·), (Q*, ·) are Group.

Abelian Group or Commutative Group: G is a group w.r.t. binary operation 'o' and it satisfies the Abelian property w.r.t. same operation 'o' then G is called Abelian Group or Commutative Group.

Example:

1. (Z, +), (R, +), (Q, +) are Abelian Group.
2. (R*, ·), (Q*, ·) are Abelian Group.

Finite and Infinite Group: If the set G contains a finite number of elements then the group (G,o) is called a finite group. Otherwise the group (G,o) is called a infinite group.

Left Cancellation Law: For any $a \neq 0$, b, c in a group G and $b = a \Rightarrow = c$

Right Cancellation Law: For any $a \neq 0$, b, c in a group G and $b\ a = c\ a \Rightarrow b = c$

Theorem: In a group G, identity element is unique.

Theorem: In a group G, inverse of any element is unique.

Show that the set of integers Z form a Group w r t the operation $*$ defined by $a * b = a + b - 1$ for all a, b in Z

Prove that the set G of rational (real) numbers other than 1, with operation such that $a \oplus b = a + b - ab$ for a, b ∊ G is an abelian group. Hence show that x = 3/2 is a solution of the $4 \oplus 5 \oplus x = 7$.

Solution: G (=Q) is the set of rational (real) numbers other than 1.

$\oplus$ is the operation consider on G as $a \oplus b = a + b - ab$ for a, b ∊ G

Show that set Q_+ of all positive rational numbers forms an abelian group under the composition defined by 'o' such that a o b = (ab)/3 for a, b ∊ Q

Show that R = { a +b√2 : a, b ∈ Q } is a commutative group w.r.t addition.

Prove that the set of matrices $A_\alpha = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$ α ∈ R forms a group w.r.t. matrix multiplication if cosθ = cosφ ⇒ θ = φ

Exercise: Show that the set G = {x/x = $2^a 3^b$ and a,b ∈ Z} is a group under multiplication.

Theorem: Let G be a group G. For a, b ∈ G, $(a\,b)^{-1} = b^{-1}a^{-1}$.

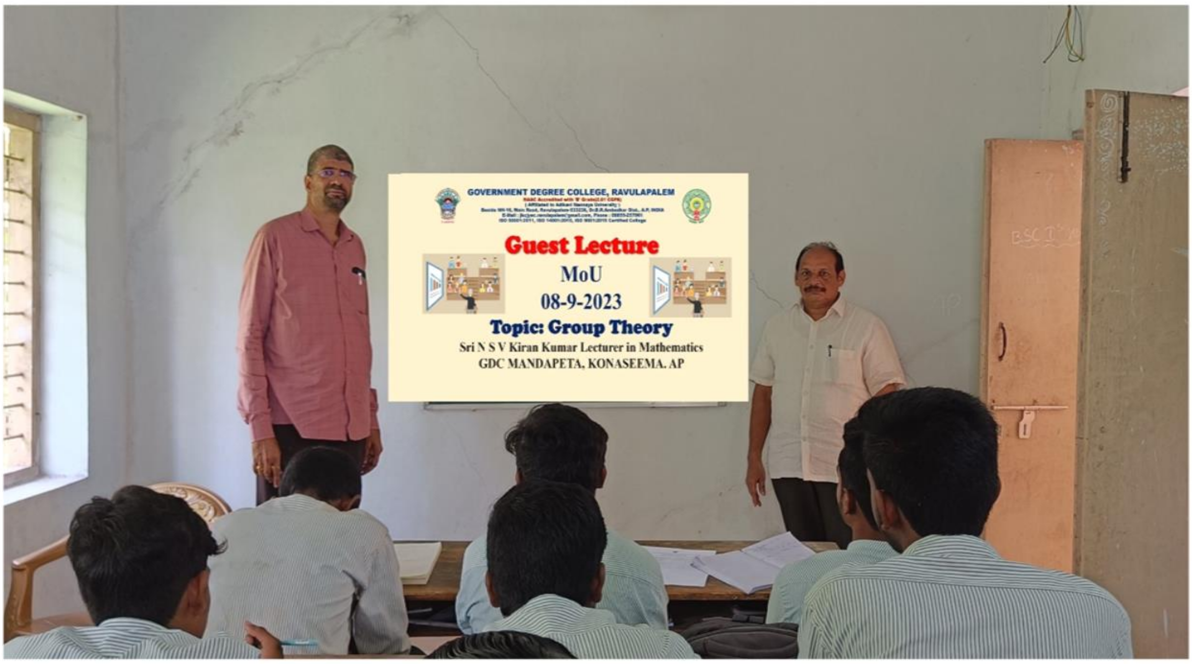
Theorem: Cancellation laws hold in a group

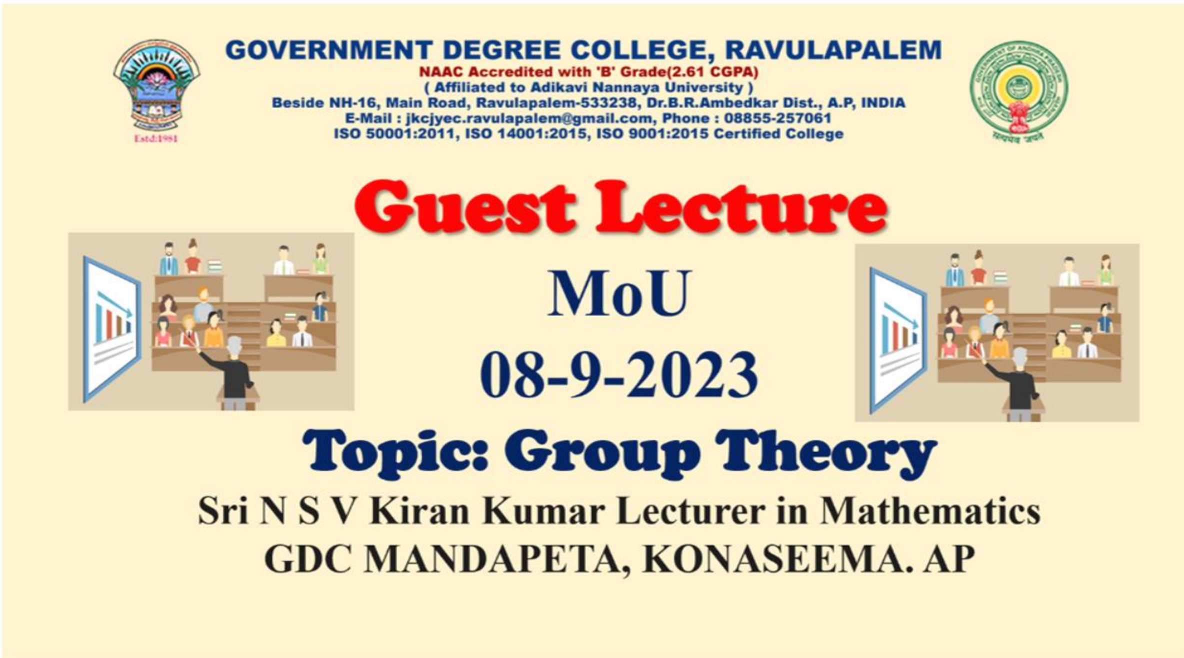
Let G be a group. Then for any $a \neq 0$, b, c ∈ G, $a\,b = a\,c \Rightarrow b = c$ (Left cancellation law) and $b\,a = c\,a \Rightarrow b = c$ (Right cancellation law)

Theorem: In a group G (≠φ), for a,b,x,y ∈ G, the equation ax = b and ya = b have unique solutions .

Prove that the set of n^{th} roots of unity under multiplication form a finite group.

Show that the set of fourth roots of unity form an abelian group w.r.t. multiplication.





PRINCIPAL
Government Degree College
Ravulapalem-533238, E.G.Dt.

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Letter for MoU Guest Lecture

From:
The principal
Government Degree College,
Ravulapalem.
B R Abedker Konaseema Dist AP

Date:05-9-2023

To,
The principal.
Government Degree College,
Mandapeta.
B R Abedker Konaseema Dist AP

Respected Sir,

I am bringing to your kind notice that our Mathematics Department was planned to conduct a MoU Guest lecture (As a part of MoU agreement with the Department of Mathematics of Government Degree College, Mandapeta.) In this context, I request you to kindly relieve on **08-9-2023** (Friday) with your respective Mathematics Lecturer **Sri N S V KIRAN KUMAR** at 10AM in MANA TV Room of our College. I request you to kindly relieve him on the above said date.

Thanking you

F/KV
8-9-23

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GOVERNMENT DEGREE COLLEGE

NAAC Accredited with 'B'Grade

Mandapeta. East godavari District , AP.

DATE: 08--09-2023.

RELIEVING LETTER

This is to certify that Sri N.S.V. KIRAN KUMAR, Lecturer in Mathematics of this college is relieved of his duties on F.N. of 08 -09-2023 to attend the Guest Lecture at Government Degree College, Ravulapalem on 08-09-2023.

PRINCIPAL

Principal
Govt. Degree College
MANDAPETA-533 308

GOVERNMENT DEGREE COLLEGE, RAVULAPALEM

(Affiliated to Adikavi Nannaya University)
Beside NH 216A, Main Road, Ravulapalem-533238, Dr.B.R.Ambedkar Konaseema Dist., A.P, INDIA
E-Mail : jkcrjyec.ravulapalem@gmail.com, Phone : 08855-257061
ISO 50001:2011, ISO 14001:2015, ISO 9001:2015 Certified College

Dt: 08-09-2023
Ravulapalem

To
The Principal
Government Degree College, Mandapeta
Kona seems Dist. AP

Attendance Certificate

This is to certify that Sri N S V Kiran Kumar, Lecturer in Mathematics, Government Degree College, Mandapeta attended as Resource Person and delivered a Guest Lecture on **"GROUP THEORY"** in II B.Sc. Class Room conducted by the Department of Mathematics, Government Degree College, Ravulapalem as a part of MoU with the college on 08-09-2023 from 10AM to 12PM.

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Government Degree Colleg
Ravulapalem-533238, E.G.D

HoD Mathematics
Government Degree College
Ravulapalem

HOD OF MATHEMATICS
Government Degree College
Ravulapalem

Principal
Government Degree College
Ravulapalem

Date :08-9-2023

Ravulapalem

Government degree College, Ravulapalem. Kona Seema dist.

Activity: MoU Guest Lecture

Resource person: K S Kiran Kumar

Lecturer in Mathematics GDC Mandapeta, Kona seem Dist.

Sl. No	Name of the student	Reg Number	Signature
1.	A.satish	221047102001	A-Satish
2	D. Ganesh	221047102002	D. Ganesh
3.	N. Durga prasad	221047102009	N. Durga prasad
4.	T. Sai Sri Ram	221047102012	T. Sai Sri Ram
5.	P.L.NARASimha	221047102011	P.L. Narasimha.
~~13~~.	~~A Satish~~	~~221047102001~~	~~A-Satish~~
6.	G.Kiran Kumar	221047102005	G.Kiran Kumar
7.	N. Harika Lakshmi	221047102006	N. Harika Lakshmi
8.	D. Lakshmi	221047102003	D. Lakshmi
9.	K. swapna	221047102008	K. swapna
10.	T. Venu madhavi	221047102013	T. Venu madhavi
11.	Y. pavani	221047102014	Y. pavani
12.	K. prasanna	221047102007	K. prasanna

Signature of the Resource person:

Signature of HoD :

Signature IQAC : Dr V. Subba...

Principal :

GUEST LECTURE

BY

Mr. B.SRINIVASA RAO
Lecturer in Mathematics
Department of Mathematics
GOVERNMENT DEGREE COLLEGE
RAVULAPALEM

TOPIC	**:**	**BASSI AND DIMENSION**
VENUE	**:**	**GOVERNMENT DEGREE COLLEGE, MANDAPETA**
		Dr. B.R. AMBEDKAR KONASEEMA DISTRICT
DATE	**:**	**February 21, 2024**
TIME	**:**	**10 AM to 12 Noon.**

TOPIC SYNOPSIS

Basis of a vector space: A sub set S of a vector space V(F) is said to be a basis of V(F) IF

1.S is linearly independent

2.L(S) = V(F) i.e, Vector space V(F) generated by S.

Example:1. S = { (1, 0, 0),(0, 1, 0),(0, 0, 1) } is a basis of the vector space $V_3(F)$.

Solution: To show that S = { (1, 0, 0),(0, 1, 0),(0, 0, 1) } is a basis of the vector space $V_3(F)$

1. S is linearly independent: $\exists$ $scalrs\ a, b, c\ \in F\ such\ that$ a(1, 0, 0) + b(0, 1, 0) + c(0, 0, 1) = O = (0, 0, 0)

$$\Rightarrow (a + 0 + 0,\ 0 + b + 0,\ 0 + 0 + c) = (0, 0, 0)$$

$$\Rightarrow (a,\ b,\ c) = (0, 0, 0)\ \Rightarrow a = 0, b = 0,\ c = 0$$

$\therefore$ S is linearly independent.

2. L(S) = $V_3(F)$: Clearly L(S) $\subseteq$ $V_3(F)$ ----- (A)

Now to show that $V_3(F) \subseteq L(S)$

For all $\alpha = (x, y, z) \in V_3(F) \Rightarrow$

$$(x, y, z) = (x, 0, 0) + (0, y, 0) + (0, 0, z)$$

$$= x(1, 0, 0) + y(0, 1, 0) + z(0, 0, 1) \in L(S)$$

$$(x, y, z) \in L(S) \Rightarrow \alpha \in L(S)$$

$$\therefore\ V_3(F) \subseteq L(S) \text{ ------(B)}$$

From (A) &(B) L(S) = $V_3(F)$

$\therefore$ S = { (1, 0, 0), (0, 1, 0), (0, 0, 1) } is a basis of the vector space $V_3(F)$

Note: { (1, 0),(0, 1) } is a basis of the vector space $V_2(F)$.

Definition: (Finite Dimensional Vector space)

A Vector space V(F) is said to be finite dimensional if there exist a finite subset S ofV(F) such that L(F) = V(F).

Example: $V_n(F)$ is finite dimensional vector space.

Definition:(Dimension of a vector space):

In a finite dimensional vector space V(F) the number of basis elements is called Dimensionof the vector space V(F) and is denoted by dim V(F).

Example: $V_n(F)$ is finite dimensional vector space and its basis

{(1, 0, 0, … . ,0), (0, 1, 0, … . ,0), … (0, 0, … . . , 1)} Number of basis elements = n

$$\dim V_n(F) = n.$$

Theorem (Existence of Basis theorem):

Every finite dimensional vector space has a basis.

Theorem (Basis extension theorem)

In a finite dimensional vector space V(F) prove that a linearly independent set is a basis or itcan be extended to form a basis of V(F).

Theorem (Invariance theorem):

If V(F) is a finite dimensional vector space then prove that any two bases have same number of elements.

Theorem: If $W_1 and\ W_2$ are two sub spaces of a finite dimensional vector space V(F) thenprove that

$$\dim(W_1 + W_2) = \dim W_1 + \dim W_2 - \dim(W_1 \cap W_2).$$

Problems:

1. Show that the set of vectors { (1, 1, 2) (1, 2, 5) (5, 3, 4) } not a basis of R^3.
2. Show that the set of vectors {(1, 2,1) (2, 1, 0) (1, -1, 2) } form a basis of R^3.
3. Show that the set of vectors {(2, 1, 0) (2, 1, 1) (2, 2, 1) } form a basis of R^3 and express the vector.

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MANDAPETA -533308 Dr. B.R Ambedkar Konaseema (dist), Andhra Pradesh

Date: 20-02-2024

From
The Principal
Government Degree College,
Mandapeta.

To
The Principal
Government Degree College,
Ravulapalem.

Sir,

Sub: Government Degree College, Mandapeta – Guest Lecture on "Basis and Dimensions" – Mr. B. Srinivasarao, Lecturer in Mathematics of your college on 21-02-2024 – Request – Regarding.

I would like to bring your kind notice that our Mathematics Department has MoU with your Mathematics Department for "Promoting and Enrichment of Teaching – Learning Process". As a part of MoU we propose to organize a Guest Lecture program on "Basis and Dimensions" on 21-02-2024. In this connection I request you to depute Mr. B.Srinivasarao, Lecturer in Mathematics of your Esteemed Institution on 21-02-2024.

Thanking you,

Yours Sincerely,

Principal
Govt. Degree College
MANDAPETA - 533 308.

Copy to
Mr. B. Srinivasarao,
Lecturer in Mathematics
GDC, Ravulapalem.

Date: 21-02-2024

ATTENDANCE CERTIFICATE

This is to certify that Mr. B. Srinivasarao, Lecturer in Mathematics, Government Degree College, Mandapeta has attended in our college on 21-02-2024 and delivered Guest Lecture on "Basis and Dimensions" for II B.Sc.(MPC & MPCS) IV Semester students of this college.

PRINCIPAL

Principal
Govt. Degree College
MANDAPETA - 533 308.

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MANDAPETA -533308 Dr. B.R Ambedkar Konaseema (dist), Andhra Pradesh

Activity : MoU Guest Lecture

Resource Person : Sri B. Srinivasarao
Lecturer in Mathematics
Government Degree College,
Ravulapalem

S.No.	Name of the Student	Reg. Number	Signature
1	N Y Padmavathi	220637101001	N. Yamuna padmavathi
2	N H V Prakash	220637101002	N. hemanth
3	R Lalitha	220637101003	R. Lalitha
4	B Mohan Krishna	220637101004	B. M. krishna
5	B Gowthami	220637101005	B. Gowthami
6	Jonna Sai Ram	220637101006	J. Sairam
7	K S B RushI	220637101007	K.S.B. Rushi
8	Makala Ramya	220637101008	M. Ramya
9	M Lakshmi	220637101009	M. Lakshmi
10	S Vardhini Devi	220637101010	S. Vardhini devi

Signature of the Resource Person

N. kiran kumar
Signature of the HOD

N.S.V. KIRAN KUMAR
LECTURER IN MATHEMATICS
GOVERNMENT DEGREE COLLEGE
MANDAPETA - 533308

Signature of the Principal

Principal
Govt. Degree College
MANDAPETA - 533 308

GOVERNMENT DEGREE COLLEGE, RAVULAPALEM
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(Affiliated to Adikavi Nannaya University, Rajahmahendravaram)
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ISO: 50001:2011, ISO 14001:2015, ISO 9001:2015 Certified College

Dr. K.JYOTHI, M.Sc. Ph.D.,
PRINCIPAL
GOVERNMENT DEGREE COLLEGE,
RAVULAPALEM, Dr.B.R.Ambedkar
Konaseema District.

Phone:08855-257061(O)
Mobile: 9440301264
Website: www.gdcrvpm.ac.in
E-Mail: jkcrjyec.ravulapalem@gmail.com

RELIEVING CERTIFICATE

Sri B.Srinivasa Rao, Lecturer in Mathematics is relieved off his duties on the A.N of 20.02.2024 to attend MOU Guest Lecture on "Basis and Dimensions" for II (B.Sc., (MPC & MPCS) IV Semester students February'2024 at Government Degree College, Mandapeta.

20/2/24

PRINCIPAL i/c
GOVERNMENT DEGREE COLLEGE
RAVULAPALEM